

# Entropy and Financial Markets

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## 1 Introduction

Entropy was first introduced as a measurement tool for disorder by Rudolf Clausius. In Greek, it means transformation. Entropy scales with the dimensionality and complexity of the system under consideration. For instance, in financial markets, it is bounded by the number of assets and the implied relations between features. In information theory, it is bounded by the channels modeled, and in physics, by the interactions of particles and degrees of freedom. Thus, it is difficult to extend entropy as a benchmark to the micro level.

To illustrate the concept based on its foundations, let us define disorder. We say a system is in disorder when its components are moving or behaving at different paces. I like to illustrate this as when we cannot predict with high certainty the next state of the system due to the volatility it experiences.

Entropy was applied by Shannon in information theory in his research paper "A Mathematical Theory of Communication," where he defined it as

$$S = - \sum p(X) \log(p(X))$$

, where  $X$  is a random variable defined on a space

$$(\Omega, \mathcal{F}, \mathbb{P}) \tag{1}$$

. This formula favors equilibrium, giving events with medium **around 37%** occurring probability the greatest contribution to the sum. Shannon entropy measures the average uncertainty across all possible outcomes in a system.

### Entropy and Market Response

When the president announces regulatory policy, the information entropy of this announcement depends on how unexpected it was. If the policy was completely anticipated, its information content

$$I = - \log(p) \tag{2}$$

approaches zero. If it was a complete surprise, the information content is high.

This high-information-content announcement then propagates through the market system, increasing market entropy. As investors reassess probabilities across all possible asset movements, the market transitions from a lower-entropy state (more predictable price paths) to a higher-entropy state (many possible outcomes become more equally likely).

The relationship between information entropy and market entropy illustrates how the dimensionality and complexity of the system amplifies the effect. A single high-information event cascades through the network of assets, market participants, and their interactions, creating system-wide entropy increase.

### **Missing Link: Why Entropy Is Overlooked in Finance**

The question I have been trying to answer is: why does nobody talk about the role of entropy in finance? In my experience, entropy has always been central to physics and chemistry. Perhaps I have not read enough articles or dove deep enough. Maybe it is because in the world of particle interactions, we can render entropy in energy terms, mostly thermal energy, as the laws of physics state:

$$\Delta S = \frac{Q}{T} \quad (3)$$

, where T is temperature and Q is thermal energy.

For instance, the golden finding every financier wishes for is a model that can predict implied volatility using, for example, Brownian motion to model the stochastic effects on option prices using realized volatility. To do this, we use the Black-Scholes equation, which is a differential equation solved using Itô's lemma:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \quad (4)$$

where V is the value of the option,  $\sigma$  is volatility, S is the price of the underlying asset, and r is usually the interest rate.

The solution for a call option is:

$$C = S_0 N(d_1) - K \exp(-rT) N(d_2) \quad (5)$$

where S is the initial price, K is the exercise price, and N is the cumulative standard normal distribution. These are modeled based on a predetermined exercise date, meaning the option cannot be exercised at any time—these are European options.